CALCULATION POLICY

Value Statement (who we are):
Coombe Hill Infants' School is an inclusive, community school with a strong tradition of mutual respect and tolerance within a nurturing family environment.
We provide an outstanding education for all children.

Vision Statement (what we strive for):
Our vision is to develop strong minds, bodies and spirit in preparation for life. We sow the seeds of curiosity, enthusiasm and resilience to ensure all children continue to delight in their lifelong love of learning.

Responsibility:Curriculum and standards Committee

Reviewed: Autumn 2020

Next review : Autumn 2023

"Safeguarding is everyone's responsibility".

Coombe Hill Infants' School complies with the relevant legal duties as set out in the Equality Act 2010 and the Human Rights Act 1998; we promote equality of opportunity and take positive steps to prevent any form of discrimination, either direct or indirect, against those with protected characteristics in all aspects of our work.

Coombe Hill Infants Calculation Policy

Author: Jack Morris (Senior Leader)
Date Created: Feb 2017
Last Reviewed: May 2023 (Sarah Gray)

Aims of this policy

This policy outlines progression through calculation strategies for addition, subtraction, multiplication and division in line with the new National Curriculum commencing September 2014. It also sets out a range of recommendations and effective practice teaching ideas for outstanding teaching and learning in maths. Through the policy, we aim to link key manipulatives (practical methods) and (visual) representations in order that the children can be vertically accelerated through each of the four strands of arithmetic. A school wide policy helps to ensure consistency of approach, enabling children to progress stage by stage through models and representations they recognise from previous teaching, allowing for deeper conceptual understanding and fluency. This policy is used as the basis for planning along with the new White Rose Maths Hub Scheme of Work. Assessment for Learning strategies are used in all year groups to identify suitable next steps and where extra consolidation practice is required. In some instances, it may be necessary to revisit previous steps before moving forward. As children move at the pace appropriate to their understanding, teachers will be presenting strategies and equipment appropriate to their level of understanding. It is expected, however, that the majority of children in each class will be working at age-appropriate levels as set out in the National Curriculum 2014 and in line with school policy.

Herts for Learning 2014, Southwark Council 2013

Rationale for KS1 curriculum

Children in Years 1 and 2 will be given a really solid foundation in the basic building blocks of mental and written arithmetic. Through being taught **place value**, they will develop an understanding of how numbers work, so that they are confident in 2-digit numbers and beginning to read and say numbers above 100. A focus on **number bonds**, first via practical hands-on experiences and subsequently using memorisation techniques, enables a good grounding in these crucial facts, and ensures that all children leave Y2 knowing the pairs of numbers which make all the numbers up to 10 at least. They will also have experienced and been taught pairs to 20. Their knowledge of number facts enables them to **add several single-digit numbers**, and to **add/subtract a single digit number to/from a 2-digit number**. Another important conceptual tool is their ability to **add/subtract 1 or 10**, and to **understand which digit changes and why**. This understanding is extended to enable children to **add and subtract multiples of ten** to and from any 2-digit number. The most important application of this knowledge is their ability to **add or subtract any pair of 2- digit numbers by counting on or back in tens and ones**. Children may extend this to adding by **partitioning numbers into tens and ones**. Children will be taught to **count in 2s, 3s, 5s and 10s**, and will have related this skill to repeated addition. They will have met and begun to learn the associated **2x, 3x, 5x and 10x tables**. Engaging in a practical way with the concept of repeated addition and the **use of arrays** enables children to develop a preliminary understanding of multiplication, and asking them to consider how many groups of a given number make a total will introduce them to the idea of division. They will also be taught to **double and halve numbers**, and will thus experience scaling up or down as a further aspect of multiplication and division. Fractions will be introduced as numbers and as operators, specifically in relation to **halves, quarters and thirds.**

Key Teaching Principles

a) Develop children's fluency with basic number facts

Fluent computational skills are dependent on understanding the composition of numbers to 10 alongside accurate rapid recall of basic number bonds to 10 then 20 as well as times-tables facts.

Spending a short time every day on these basic facts quickly leads to improved fluency. Using materials from the NCETM Mastering Number programme we are committed to ensuring our children develop fluency, number sense, can communicate their ideas and have confidence as Mathematicians. The programme allows children to recognise patterns and relationships within and between numbers.

b) Develop children's fluency in mental calculation

We recognise the importance of the mental strategies and known facts that form the basis of all calculations. **Children must develop a good mental map of the open number line and be able to visualise where numbers are located**. Knowledge of proximity of numbers enables children to make choices between counting up and counting back when subtracting for example. Knowing where numbers lie between the multiples of tens helps bridging. The following check lists outline the key skills and number facts that children are expected to develop throughout the school.

To add and subtract successfully, children should be able to:

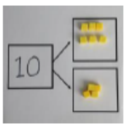
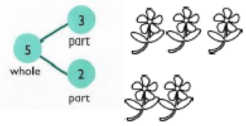
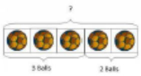
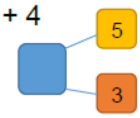
· recall all addition pairs to 9 + 9 and number bonds to 20
· recognise addition and subtraction as inverse operations
· add mentally a series of one digit numbers (e.g. 5 + 8 + 4)
· add and subtract multiples of 10 or 100 using the related addition fact and their knowledge of place value (e.g. 600 + 700, 160 — 70)
· partition 2 and 3 digit numbers into multiples of 100, 10 and 1 in different ways (e.g. partition 74 into 70 + 4 or 60 + 14)
· use estimation by rounding to check answers are reasonable

To multiply and divide successfully, children should be able to:

· add and subtract accurately and efficiently
· recall multiplication and division facts for 2x, 5x and 10x tables
· use multiplication/division facts to estimate how many times one number divides into another
· know the outcome of multiplying by 0 and by 1 and of dividing by 1
· derive other results from multiplication and division facts and multiplication and division by 10
· notice and recall with increasing fluency inverse facts
· partition numbers into 100s, 10s and 1s
· understand how the principles of commutative, associative and distributive laws apply or do not apply to multiplication and division
· investigate and learn rules for divisibility

c) Move from the concrete to the visual and the abstract (CPA Model)

Children's conceptual understanding and fluency is strengthened if they experience concrete, visual and abstract representations of a concept during a topic or lesson. Moving between the concrete and the abstract helps children to connect abstract symbols with familiar contexts, thus providing the opportunity to make sense of, and develop fluency in the use of, abstract symbols. For example:

Objective and Strategies	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model	  Use cubes to add two numbers together as a group or in a bar. 	  Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7 10= 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.

J. Knapp 2016

d) Anticipate difficult points

Difficult points or **common misconceptions need to be identified and anticipated** when lessons are being designed and these need to be **an explicit part of the teaching**, rather than the teacher just responding to children's difficulties if they happen to arise in the lesson. The teacher should be actively seeking to uncover possible difficulties because if one child has a difficulty it is likely that others will have a similar difficulty.

Difficult points also give an opportunity to reinforce that we learn most by working on and through ideas with which we are not fully secure or confident. Discussion about difficult points can be stimulated by asking children to share thoughts about their own examples when these show errors arising from insufficient understanding.

e) Use questioning to develop mathematical reasoning

Despite concrete and visual representations, understanding of mathematical concepts does not happen automatically. Children need to reason by themselves and make their own connections. Getting children into good habits from Year 1, in terms of reasoning and looking for pattern and connections in the mathematics, is important. The question "What's the same, what's different?" can be used frequently to make comparisons.

Teachers' questions in mathematics lessons are often asked in order to find out whether children can give the right answer to a calculation or a problem. But **in order to develop children's**

conceptual understanding and fluency there needs to be a strong and consistent focus on questioning that encourages and develops their mathematical reasoning.

This can be done simply by asking children to explain 'how they worked out a calculation or solved a problem', and 'to compare and contrast different methods' that are described. Thus children quickly come to expect that they need to explain and justify their mathematical reasoning, and they soon start to do so automatically – and enthusiastically. Provide as much scaffolding as the children require.

? Rich questioning strategies include:

- "What's the same, what's different?" For example, discussion of the variation in these calculations can help children to identify the relationship between the calculations and hence to use the pattern to calculate the answers.

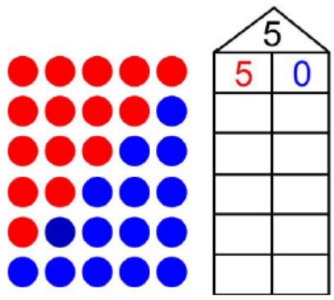
23 + 10	23 + 20	23 + 30	23 + 40

- "Odd one out". Which is the odd one out in this list of numbers: 24, 15, 16 and 22? This encourages children to apply their existing conceptual understanding.
- "Here's the answer. What could the question have been?". Children are asked to suggest possible questions that have a given answer.
- "True or False: 13 + 20 = 24". Give your reasoning; "I know this because…"
- "Which questions are easy/hard? 23 + 10, 53 + 10, 54 + 1, 54 + 9." Explain why.
- Fact families: "Which 4 calculations link these numbers; 12 15 3?"
- "What else do you know? If you know 2 + 8 = 10…" (then 20 + 80 = 100)
- Missing number/symbol questions
- Making an estimate: which calculation has an answer between 10 and 20…5 + 16, 3 + 9…
- "Numbers in 5x table are odd. Is this always, sometimes or never true?"

NCETM 2015

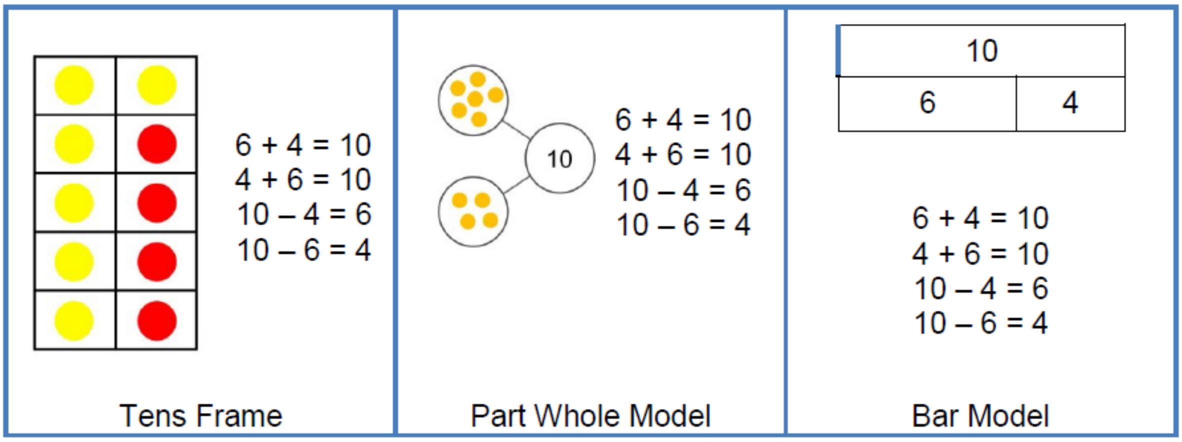
f) Expose mathematical structure and work systematically

Developing instant recall alongside conceptual understanding **of number bonds to 20 is important**. This can be supported through the use of images such as:



Shanghai Textbook Grade 1 (aged 6/7)

The image lends itself to seeing patterns and working systematically and children can connect one number fact to another and be certain when they have found all the bonds to 5. Using other structured models such as tens frames, part whole models or bar models can help children to reason about mathematical relationships.



Connections between these models should be made, so that children understand the same mathematics is represented in different ways. Asking the question “What’s the same what’s different?” has the potential for children to draw out the connections.

Illustrating that the same structure can be applied to any numbers helps children to generalise mathematical ideas and build from the simple to more complex numbers, recognising that the structure stays the same; it is only the numbers that change.

g) Develop children’s understanding of the = symbol

The = symbol is an assertion of equivalence. If we write:

$$3 + 4 = 6 + 1$$

then we are saying that what is on the left of the = symbol is necessarily equivalent to what is on the right of the symbol. But many children interpret = as being simply an instruction to evaluate a calculation, as a result of always seeing it used thus:

$$3 + 4 = \qquad 5 \times 7 = \qquad 16 - 9 =$$

If children only think of = as meaning “work out the answer to this calculation” then they are likely to get confused by empty box questions in Year 2 such as:

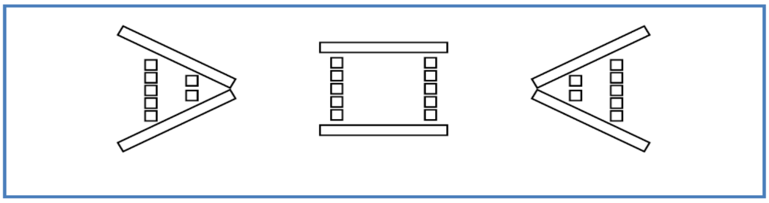
$$3 + \square = 8$$

One way to model equivalence such as 2 + 3 = 4 + 1 is to use balance scales.

NCETM 2015

h) Teach inequality alongside teaching equality

To help young children develop their understanding of equality, they **also need to develop understanding of inequality through learning about inequality** before, or at the same time as, equality. One way to introduce the < and > signs is to use dienes to make a concrete and visual representations such as:



to show 5 is greater than 2 (5 > 2), 5 is equal to 5 (5 = 5), and 2 is less than 5 (2 < 5).

NCETM 2015

i) Use empty box problems

Empty box problems are a powerful way to help children develop a strong sense of number through intelligent practice. They **provide the opportunity for reasoning and finding easy ways to calculate**. They enable children to practise procedures, whilst at the same time thinking about conceptual connections.
A sequence of examples such as

3 + □ = 8
3 + □ = 9
3 + □ = 10
3 + □ = 11

helps children develop their understanding that the = symbol is an assertion of equivalence, and invites children to spot the pattern and use this to work out the answers. Children should also be given examples where the empty box represents the operation.

NCETM 2015

j) Challenge thoughtfully - variation

Insufficient challenge can lead to boredom and poor behaviour. By doing a pre-assessment before a new topic is due to commence, teaching can be better pitched and the needs of the children can be better met. **Subtle variation is a key strategy**. Present problems in different ways which are underpinned by known understanding, whereby only 1 variable is changed. This allows connections to be made whilst reinforcing the conceptual understanding.

Non-standard, non-routine, interconnected, inventive activities allow children to apply their knowledge and understanding in a new context. This is Working at Greater Depth, and deeper learning occurs when children are enjoying being stuck, being resilient and learning from their mistakes. A **Mastery Curriculum** promotes an ever-growing development of understanding within children. It **is not a to-do list of knowledge, but a way of being** – the skills and attitudes to succeed in maths.

C. Quigley 2016

Key Learning Principles

a) Expect children to use correct mathematical terminology and to express their reasoning in complete sentences

The quality of children's mathematical reasoning and conceptual understanding is significantly enhanced if they are consistently expected to **use correct mathematical terminology and to explain their mathematical thinking in complete sentences**.
Teachers can model a sentence stem for children to communicate their ideas with mathematical precision and clarity. For example: "When adding 10 to a number, the ones digit stays the same" This is repeated in chorus using the same sentence, which helps to embed the concept. Back this up with a visual classroom display.

Other agreed vocabulary to use:

a) Do not use "equals" but "is equal to" or "is the same as"
b) The terms "parts" and "whole" should be introduced with addition and subtraction to support transition from concrete/visual strategies to abstract
c) "ones" not "units" as this better relates to diene values and can be confused with measurement
d) "zero" not "oh/nought"
e) "digit" rather than "number" when discussing place value. E.g. in '23', 2 is a digit worth 2 tens or 20

b) Contextualise the mathematics

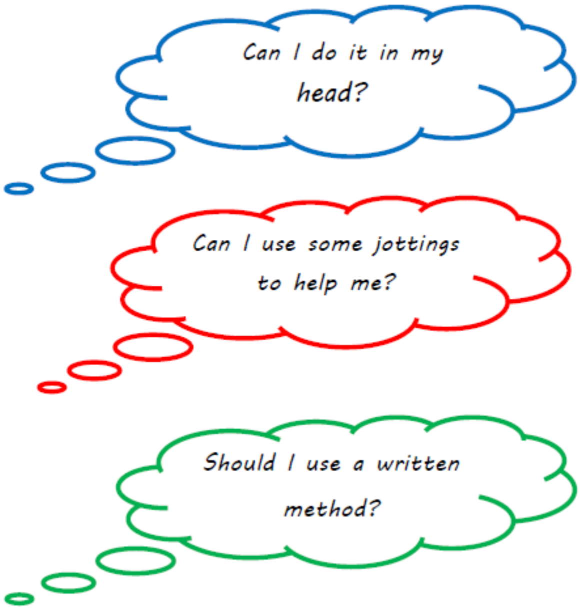
Relate the maths to real life situations. **Storytelling is an effective vehicle for introducing mathematical concepts/vocabulary**. A lesson about addition and subtraction could start with this contextual story: "There are 11 people on a bus. At the next stop 4 people get on. At the next stop 6 people get off. How many are now on the bus?" This helps children develop their understanding of the concepts of addition and subtraction. But during the lesson the teacher should keep returning to the story.

This can be part of the daily register routine, referring to the number of meals the school needs to make considering absences and lunchbox numbers. NCETM 2015

c) Hierarchy of thinking

As part of a child's learning in calculation, **they need to be taught how to select the best method according to the numbers**, with an emphasis on mental arithmetic. Encourage children to give an approximate answer and check it using practical or written methods.

The hierarchy of thinking should be:



Cliffe VC Primary School, 2014

d) Don't count, calculate - Magic 10!

Young children benefit from being helped at an early stage to start calculating, rather than relying on 'counting on' as a way of calculating. For example, with a sum such as:

$$4 + 7 =$$

Rather than starting at 4 and counting on 7, children could use their knowledge and bridge to 10 to deduce that because 4 + 6 = 10, so 4 + 7 must equal 11.

In particular is the importance of 10 and partitioning numbers to bridge through 10. For example:

$$9 + 6 \quad = \quad 9 + 1 + 5 \quad = \quad 10 + 5 \quad = \quad 15$$

Shanghai (model) teachers refer to “magic 10”. It is helpful to make a 10 as this makes the calculation easier.

NCETM 2015

Maths Mastery teaching

Maths Mastery teaching at Coombe Hill Infants involves supporting teachers to:

- **know** the **Early Years/National Curriculum** for their year group very well and know how to find out what children are expected to have learned in the preceding year.
- **understand** the importance of a **common vocabulary** across the school.
- **aim** for 3-5 minutes of **mental arithmetic** at the start of every lesson/every day.
- **use** planning based on the **Concrete-Pictorial-Abstract approach** across a learning theme.
- **include 1) fluency practise 2) reasoning practise and 3) problem solving** across each learning theme.
- **strive** to develop an understanding of the **small steps of progression** through arithmetic, referring to progression steps in this policy, with the support of the Maths Co-ordinator/external training.
- **share** the **hierarchy of thinking** with children which encourages independence.

Resource progression

Year Group	Models and Resources		Written Methods	Misconceptions
EYFS	● Objects for counting (topic related) ● Physical jumps ● Counting Stick ● 1-10 numberline ● Pegboards ● Number Fans	● Numicon ● hands ● pegs on hanger ● beads ● snakes and ladders ● rulers	● + and – ● combining 2 groups ● pictures of objects	● Language confusion ● confusing addition and subtraction
Year 1	● Counters ● Numicon ● No. line lrg + sml ● 100 square ● Other no. grids ● dominoes ● fingers ● partitioning ● arrow cards ● dienes	● children ● unifix ● beads ● buttons ● pegs on hanger ● bundles of 10 straws ● pictures ● jumping numberline	● No. lines ● Pictorial ● Whiteboards ● Column Add for HA ● closed numberline ● bar model using cubes ● addition over 10s boundary ● pictograms	● Miscounting objects ● Starting on wrong number ● Don’t stop counting on/back, just keep going ● language

	• hands • counting objects • arrays + squared paper • cubes	• coins • bar model • lego bricks • bean bags • money tin – loud counting	• number bonds	
Year 2	• Numberlines • Numicon • Counting beads • Counting stick • 100 square • Open numberline • Dienes • Coins • Human number line • Click, clap, stamp	• Coin tin • cubes • digit cards • dice • blank number frame • arrow cards • hands/fingers • Cuisenaire • Number grids (red/yellow)	• arrays/pictures • bar modelling • open numberline • word problems • bar graphs	• Saying 1 on number they start on for subtraction so answer is 1 out • inverse operations • missing numbers • bridging over 10 • language

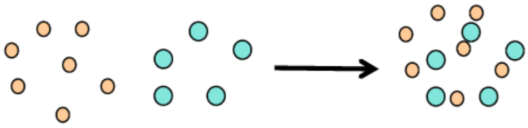
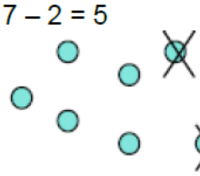
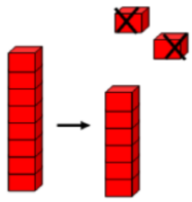
Click here for **CPA: Concrete, PIctorial, Abstract Methods in Maths** **if reading this policy online, else see overleaf.**

A) Progression in addition and subtraction

Addition and subtraction are connected.

part	part
whole	

Addition names the **whole** in terms of the **parts** and subtraction names a missing **part** of the **whole**.

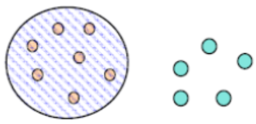
<table>
<tr><th><u>Addition</u></th><th><u>Subtraction</u></th></tr>
<tr><td><u>Combining two sets (aggregation)</u>
Putting together – two or more amounts or numbers are put together to make a total
7 + 5 = 12

Count one set, then the other set. Combine the sets and count again. Starting at 1.
Counting along the bead bar, count out the 2 sets, then draw them together, count again. Starting at 1.
</td><td><u>Taking away (separation model)</u>
Where one quantity is taken away from another to calculate what is left.
7 – 2 = 5

Multilink towers - to physically take away objects.
</td></tr>
</table>

<u>Combining two sets (augmentation)</u>

This stage is essential in starting children to calculate rather than counting

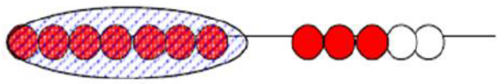
Where one quantity is increased by some amount. Count on from the total of the first set, e.g. put 3 in your head and count on 2. Always start with the largest number.

<u>Counters:</u>



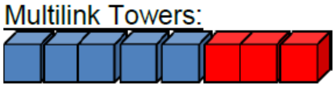
Start with 7, then count on 8, 9, 10, 11, 12

<u>Bead strings:</u>

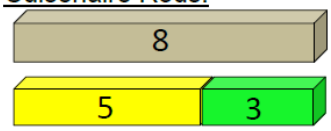


Make a set of 7 and a set of 5. Then count on from 7.

<u>Multilink Towers:</u>



<u>Cuisenaire Rods:</u>



Number tracks:

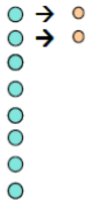
Start on 5 then count on 3 more

<u>Finding the difference (comparison model)</u>

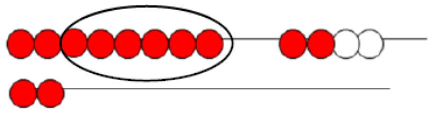
Two quantities are compared to find the difference.

8 – 2 = 6

<u>Counters:</u>

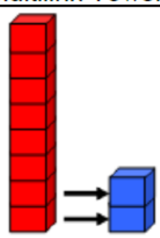


<u>Bead strings:</u>

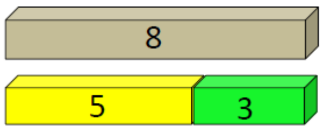


Make a set of 8 and a set of 2. Then count the gap.

<u>Multilink Towers:</u>



<u>Cuisenaire Rods:</u>



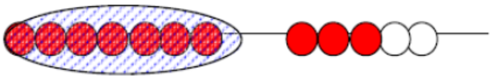
Number tracks:

Start with the smaller number and count the gap to the larger number.

Bridging through 10s

This stage encourages children to become more efficient and begin to employ known facts.

Bead string:

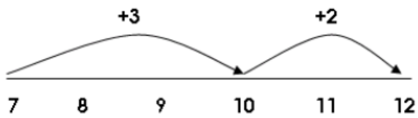


7 + 5 is decomposed / partitioned into 7 + 3 + 2. The bead string illustrates 'how many more to the next multiple of 10?' (children should identify how their number bonds are being applied) and then 'if we have used 3 of the 5 to get to 10, how many more do we need to add on? (ability to decompose/partition all numbers applied)

Number track:

Steps can be recorded on a number track alongside the bead string, prior to transition to number line.

Number line

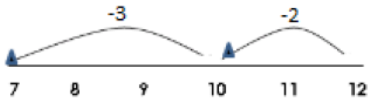


Bead string:

12 – 7 is decomposed / partitioned in 12 – 2 – 5. The bead string illustrates 'from 12 how many to the last/previous multiple of 10?' and then 'if we have used 2 of the 7 we need to subtract, how many more do we need to count back? (ability to decompose/partition all numbers applied)

Number Track:

Steps can be recorded on a number track alongside the bead string, prior to transition to number line.



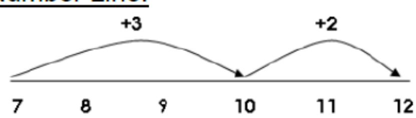
Counting up or 'Shop keepers' method

Bead string:

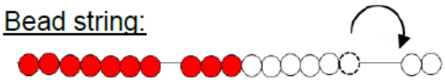
12 – 7 becomes 7 + 3 + 2.
Starting from 7 on the bead string 'how many more to the next multiple of 10?' (children should recognise how their number bonds are being applied), 'how many more to get to 12?'.

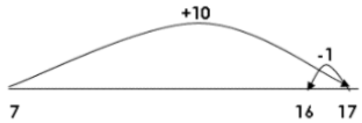
Number Track:

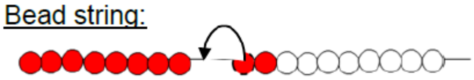
Number Line:

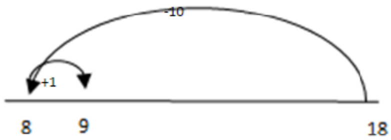


<table>
<tr><td colspan="2"><u>Compensation model (adding 9 and 11) (optional)</u>
This model of calculation encourages efficiency and application of known facts (how to add ten)</td></tr>
<tr><td>7 + 9

<u>Bead string:</u>

Children find 7, then add on 10 and then adjust by removing 1.

<u>Number line:</u>

</td><td>18 – 9

<u>Bead string:</u>

Children find 18, then subtract 10 and then adjust by adding 1.

<u>Number line:</u>

</td></tr>
</table>

Working with larger numbers

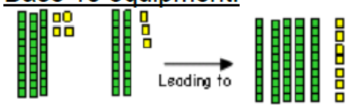
Tens and ones + tens and ones

Ensure that the children have been transitioned onto Base 10 equipment and understand the abstract nature of the single 'tens' sticks and 'hundreds' blocks

Partitioning (Aggregation model)

34 + 23 = 57

Base 10 equipment:

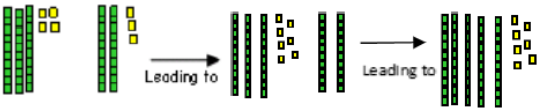


Children create the two sets with Base 10 equipment and then combine; ones with ones, tens with tens.

Partitioning (Augmentation model)

Base 10 equipment:

Encourage the children to begin counting from the first set of ones and tens, avoiding counting from 1. Beginning with the ones in preparation for formal columnar method.



Number line:

At this stage, children can begin to use an informal method to support, record and explain their method. (optional)

30 + 4 + 20 + 3

50 7

57

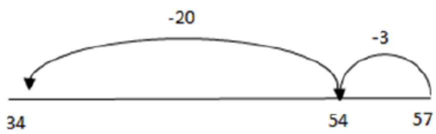
Take away (Separation model)

57 – 23 = 34

Base 10 equipment:

Children remove the lower quantity from the larger set, starting with the ones and then the tens. In preparation for formal decomposition.

Number Line:



At this stage, children can began to use an informal method to support, record and explain their method (optional)

(50 + 7) - (20 + 3)

30 4

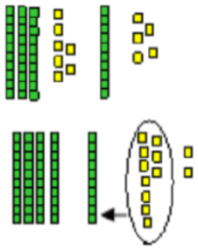
34

Bridging with larger numbers

Once secure in partitioning for addition, children begin to explore exchanging. What happens if the ones are greater than 10? Introduce the term 'exchange'. Using the Base 10 equipment, children exchange ten ones for a single tens rod, which is equivalent to crossing the tens boundary on the bead string or number line.

Base 10 equipment:

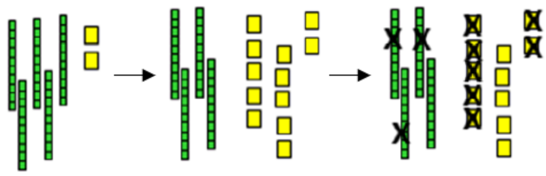
37 + 15 = 52



Discuss counting on from the larger number irrespective of the order of the calculation.

Base 10 equipment:

52 – 37 = 15

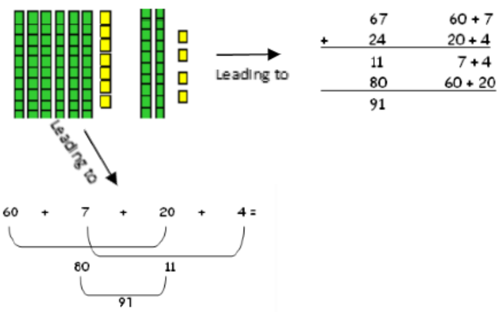


Expanded Vertical Method (optional)

Children are then introduced to the expanded vertical method to ensure that they make the link between using Base 10 equipment, partitioning and recording using this expanded vertical method.

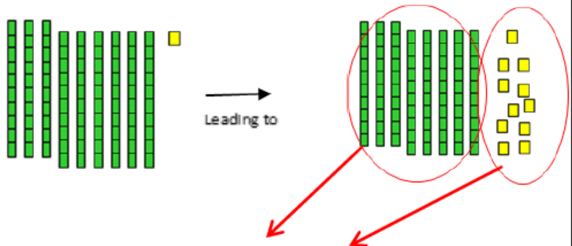
Base 10 equipment:

67 + 24 = 91



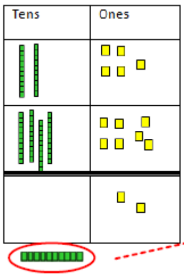
Base 10 equipment:

91 – 67 = 24



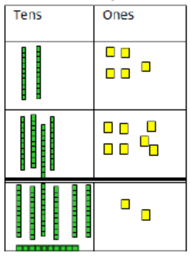
80 11
~~90~~ + ~~1~~
\- 60 + 7
20 + 4

Compact method



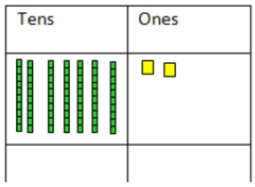
$$\begin{array}{r} 25 \\ 47 \\ \hline 2 \\ \hline {\scriptstyle 1}\;\; \end{array}$$

Leading to



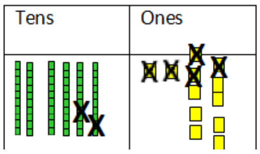
$$\begin{array}{r} 25 \\ 47 \\ \hline 72 \\ \hline {\scriptstyle 1}\;\; \end{array}$$

Compact decomposition



$$\begin{array}{r} 72 \\ -25 \\ \hline 47 \\ \hline \end{array}$$

$$\begin{array}{r} {}^{6}\cancel{7}\,{}^{1}2 \\ -25 \\ \hline \\ \hline \end{array}$$



$$\begin{array}{r} {}^{6}\cancel{7}\,{}^{1}2 \\ -25 \\ \hline 47 \\ \hline \end{array}$$

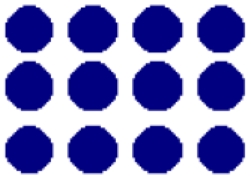
B) Progression in Multiplication and Division

Multiplication and division are connected.

part	part	part	part
whole			

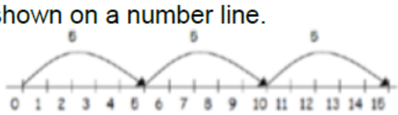
Both express the relationship between a number of equal **parts** and the **whole**.

The following array, consisting of four columns and three rows, could be used to represent the number sentences:

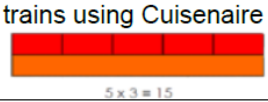
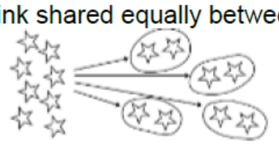


3 x 4 = 12	12 ÷ 4 = 3
4 x 3 = 12	12 ÷ 3 = 4
3 + 3 + 3 + 3 = 12	12 – 4 – 4 – 4 = 0
4 + 4 + 4 = 12	12 – 3 – 3 – 3 – 3 = 0

<table>
<tr><th>Multiplication</th><th>Division</th></tr>
<tr><td><u>Early experiences</u>
Children will have real, practical experiences of handling equal groups of objects and counting in 2s, 10s and 5s. Children work on practical problem solving activities involving equal sets or groups.
 

</td><td>Children will understand equal groups and share objects out in play and problem solving. They will count in 2s, 10s and 5s.
</td></tr>
<tr><td rowspan="2"><u>Repeated addition (repeated aggregation)</u>
3 times 5 is 5 + 5 + 5 = 15 or 5 lots of 3 or 5 x 3
Children learn that repeated addition can be shown on a number line.


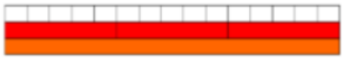
Children learn that repeated addition can be shown on a bead string.

Children also learn to partition totals into equal trains using Cuisenaire Rods

</td><td><u>Sharing equally</u>
6 sweets get shared between 2 people. How many sweets do they each get? A bottle of fizzy drink shared equally between 4 glasses.
</td></tr>
<tr><td><u>Grouping or repeated subtraction</u>
There are 6 sweets. How many people can have 2 sweets each?
</td></tr>
</table>

Scaling

This is an extension of augmentation in addition, except, with multiplication, we increase the quantity by a scale factor not by a fixed amount. For example, where you have 3 giant marbles and you swap each one for 5 of your friend's small marbles, you will end up with 15 marbles. This can be written as:

1 + 1 + 1 = 3 □ scaled up by 5 □5 + 5 + 5 = 15

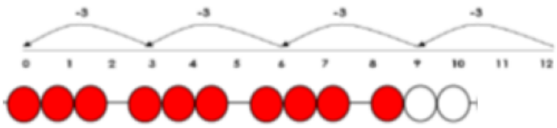


For example, find a ribbon that is 4 times as long as the blue ribbon.

We should also be aware that if we multiply by a number less than 1, this would correspond to a scaling that reduces the size of the quantity. For example, scaling 3 by a factor of 0.5 would reduce it to 1.5, corresponding to 3 x 0.5 = 1.5.

Repeated subtraction using a bead string or number line

12 ÷ 3 = 4



The bead string helps children with interpreting division calculations, recognising that 12 ÷ 3 can be seen as 'how many 3s make 12?'

Cuisenaire Rods also help children to interpret division calculations.

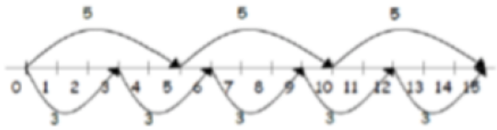
Commutativity

Children learn that 3 x 5 has the same total as 5 x 3.

This can also be shown on the number line.

3 x 5 = 15

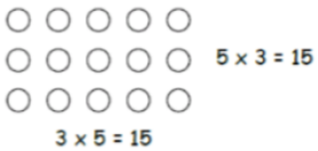
5 x 3 = 15



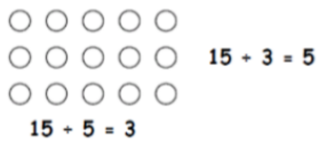
Children learn that division is **not** commutative and link this to subtraction.

Arrays

Children learn to model a multiplication calculation using an array. This model supports their understanding of **commutativity** and the development of the grid in a written method. It also supports the finding of factors of a number.



Children learn to model a division calculation using an array. This model supports their understanding of the development of partitioning and the 'bus stop method' in a written method. This model also connects division to **finding fractions** of discrete quantities.



Inverse operations

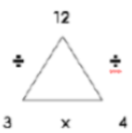
Trios can be used to model the 4 related multiplication and division facts. Children learn to state the 4 related facts.

3 x 4 = 12

4 x 3 = 12

12 ÷ 3 = 4

12 ÷ 4 = 3



Children use symbols to represent unknown numbers and complete equations using inverse operations. They use this strategy to calculate the missing numbers in calculations.

□ x 5 = 20   3 x Δ = 18   O x □ = 32

24 ÷ 2 = □   15 ÷ O = 3   Δ ÷ 10 = 8

This can also be supported using arrays: e.g. 3 X ? = 12